# Digital system to classify images by Bessel masks

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## 1. Introduction

The pattern recognition digital systems based on correlations have been studied for several years, their applications are broad and varied. For example, such systems are used to classify microorganisms as bacteria, chromosomes, infected cells, etc. Also, they are used to identify automobile brands and plates, bone structures, digital fingerprints, among other things [1]. For humans it is relatively easy to identify a fragment of some object if it is isolate and maybe this identification could be more complicated if it is partially overlapped by other object. It is not a trivial issue to reproduce this human ability digitally.

In this paper a digital system to identify fragments of objects is presented. This digital system is invariant to position, rotation, noise and non-homogenous illumination.

## 2. The system

The system classifies grey-scale images of $n \times n$ pixels. In an easy manner, the invariant to translation is achieved working with the modulus of the Fourier transform of the image $I$ ($|\mathcal{F}(I)|$).

*The Bessel masks*

To obtain the invariant to rotation, binary masks of concentric rings are built using the ratio of the Bessel function of first kind and first order by its argument, that is,

$$y(x) = \begin{cases} \frac{J_1(x-c_x)}{x-c_x}, & \text{if } x \neq c_x, \\ 1, & \text{if } x = c_x, \end{cases} \tag{1}$$

where $x = 1, \ldots, n$ and $(c_x, c_x)$ is the center-pixel of the image. Based on Eq. (1), the following binary functions are set,

$$Z_{_P}(x) = \begin{cases} 1, & \text{si } y(x) > 0, \\ 0, & \text{si } y(x) \leq 0, \end{cases} \qquad Z_{_N}(x) \;\; = \;\; \begin{cases} 1, & \text{si } y(x) \leq 0, \\ 0, & \text{si } y(x) > 0. \end{cases} \tag{2}$$

The functions $Z_{_P}(x)$ and $Z_{_N}(x)$ are rotated 180°, according to the rotation-axis $x = c_x$, to get concentric cylinders of height one and different widths, all of them centered in $(c_x, c_x)$. Taking a cross-section in each set of those cylinders, the Bessel masks $B_{_P}$ and $B_{_N}$ are generated, Fig. **1**a and b [2, 3].

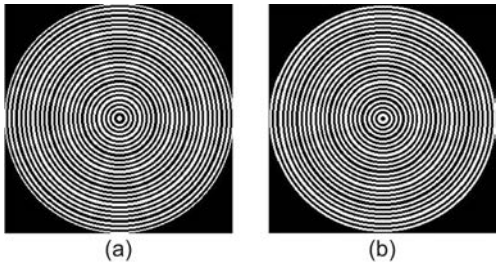


**Figure 1**: (a) Bessel mask $B_P$. (b) Bessel mask $B_N$.

*The signature of the image*

The signature of a given image, for example Fig. **2**a, is obtained by filtering its $|\mathcal{F}(I)|$ (Fig. **2**b) by a Bessel mask (in this example $B_P$ is used, Fig. **2**c), that is $H_P = B_P * |\mathcal{F}(I)|$ , where $*$ means a point to point multiplication of both images (Fig. **2**d). Then, the rings in $H_P$ are enumerate from inside to outside to build the following set,

$$index = \{ring\ index \in \bar{n}\}, \tag{3}$$

where $\bar{n} = \{1, \ldots, n : n \in \mathbb{N}\}$. Next, the intensity values in each ring are adding to get the signature of the image, that is

$$\begin{aligned} signature &= index \to A \subset \mathbb{R}, \\ signature(ring\,index) &= \sum H_P, \text{ if } H_P(x,y) \text{ are in the ring } ring\,index. \end{aligned} \tag{4}$$

In Fig. **2**e the graph of the *signature* of Fig. **2**a is presented.

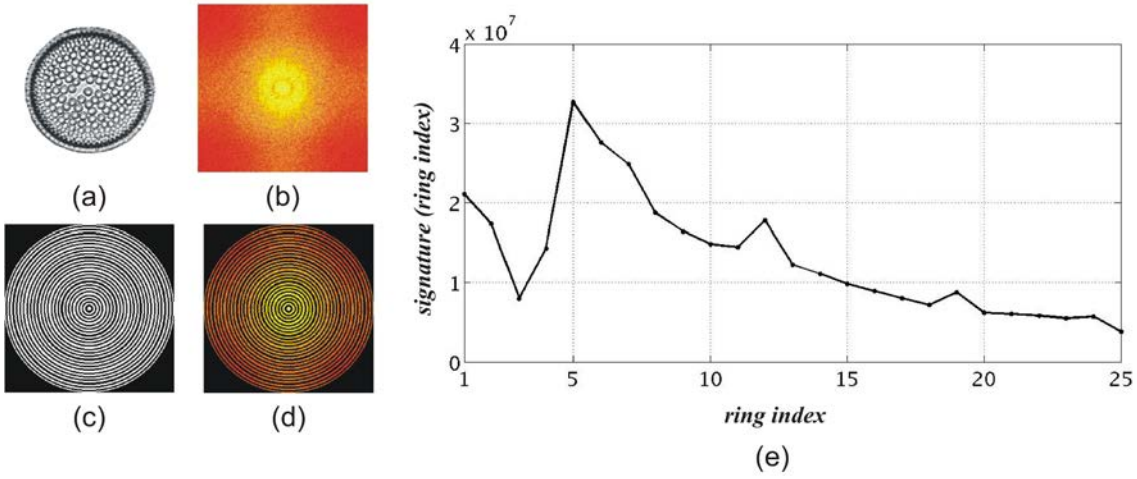


**Figure 2**: (a) $I$: *Actinocyclus ingens - Rattray.* (b) $|\mathcal{F}(I)|$. (c) Bessel Mask $B_P$. (d) $H_P = B_P * |\mathcal{F}(I)|$, where $*$ is a point to point multiplication of the images. (e) Signature of Fig. **2**a.

## 3. Classification

To perform the recognition of patterns, the first step is set the two signatures of each images in the target images data base, $\beta_R = \left\{R_j \in M_{n\times n} :\ j = 1, \ldots, k;\ k \in \mathbb{N}\right\}$. Lets call those signatures as

$S^{P}_{R_j}$ y $S^{N}_{R_j}$, the superindex $P$ means that the signature was obtained using $B_P$ and the superindex $N$ represents the signature that came from $B_N$. The signatures are weighted as $\tilde{S}^{\alpha}_{R_j} = \eta^{\alpha}_{R_j} S^{\alpha}_{R_j}$, $\alpha = P$ or $N$, and $\eta^{\alpha}_{R_j} = max\left\{\left|C_L(S^{\alpha}_{R_j})\right|\right\}$, which is the maximum value of the linear autocorrelation calculated from the linear correlation equation,

$$C_L(S_1, S_2) = \mathcal{F}^{-1}\left\{|\mathcal{F}(S_2)|\, e^{i\phi}\, |\mathcal{F}(S_1)|\, e^{-i\varphi}\right\}, \tag{5}$$

here $\varphi$ and $\phi$ are the phases of the Fourier transform of $S_1$ and $S_2$, respectively. Finally, the pattern in the target image $R_j$ is characterised by

$$r_{R_j} = \frac{\max\left\{C_L(\tilde{S}^{P}_{R_j})\right\}}{(N^P-1)\sigma^2_{\tilde{S}^{P}_{R_j}}} \frac{\max\left\{C_L(\tilde{S}^{N}_{R_j})\right\}}{(N^N-1)\sigma^2_{\tilde{S}^{N}_{R_j}}}, \tag{6}$$

with $N^{\alpha}$ as the cardinality of the domain of $\tilde{S}^{\alpha}_{R_j}$ and $\sigma_{\tilde{S}^{\alpha}_{R_j}}$ is its standar deviation. Now, the pattern in a problem image $P$, is set by

$$r_P = \frac{\max\left\{C_L(\tilde{S}^{P}_{R_j}, \tilde{S}^{P}_{P})\right\}}{(N^P-1)\sigma_{\tilde{S}^{P}_{R_j}}\,\sigma_{\tilde{S}^{P}_{P}}} \frac{\max\left\{C_L(\tilde{S}^{N}_{R_j}, \tilde{S}^{N}_{P})\right\}}{(N^N-1)\sigma_{\tilde{S}^{N}_{R_j}}\,\sigma_{\tilde{S}^{N}_{P}}}. \tag{7}$$

where $\tilde{S}^{\alpha}_{P}$ and $\sigma_{\tilde{S}^{N}_{P}}$ are the weighted signature and the standard deviation of the problem image. If $r_P$ is similar to $r_{R_j}$, therefore $P$ and $R_j$ are the same, otherwise they are different.

Due to $R_j$ could be presented in different rotation angles, the system will be trained using 360 rotated images of $R_j$ ($\Delta\theta = 1°$) and their corresponding signatures $r^{\theta}_{R_j}$ were obtained. Because of $\left\{r^{\theta}_{R_j}, \theta = 0, 1, \ldots, 359\right\}$ does not have a normal distribution, those values are normalised by the $Z$-Fisher transform to get the confidence interval for the correlation values.

The $Z$-Fisher value for $r^{\theta}_{R_j}$ is given by $Z_{r^{\theta}_{R_j}} = 1.1513\; ln\left(\frac{1+r^{\theta}_{R_j}}{1-r^{\theta}_{R_j}}\right)$. Thus, the 99% confidence interval for $Z_{r^{\theta}_{R_j}}$ is given by $\left[Z^{-}_{r^{\theta}_{R_j}}, Z^{+}_{r^{\theta}_{R_j}}\right] = \left[Z_{r^{\theta}_{R_j}} - 2.575\,\sigma_Z,\; Z_{r^{\theta}_{R_j}} + 2.575\,\sigma_Z\right]$, with a standard deviation of $\sigma_Z = \frac{1}{\sqrt{n-3}}$ and $n = 360$ the size of the sample. Hence, the confidence interval for the correlation coefficient $\rho_{r^{\theta}_{R_j}}$ is $\rho^{-}_{r^{\theta}_{R_j}} \leq \rho_{r^{\theta}_{R_j}} \leq \rho^{+}_{r^{\theta}_{R_j}}$, where

$$\rho^{-}_{r^{\theta}_{R_j}} = \frac{\exp\left(2Z^{-}_{r^{\theta}_{R_j}}\right) - 1}{\exp\left(2Z^{-}_{r^{\theta}_{R_j}}\right) + 1}, \qquad \rho^{+}_{r^{\theta}_{R_j}} = \frac{\exp\left(2Z^{+}_{r^{\theta}_{R_j}}\right) - 1}{\exp\left(2Z^{+}_{r^{\theta}_{R_j}}\right) + 1}. \tag{8}$$

From $R_j$ and their rotated images there are 360 values of $\rho^{-}_{r^{\theta}_{R_j}}$ and another 360 values for $\rho^{+}_{r^{\theta}_{R_j}}$, then the confidence interval of 99% to decide if a problem image and $R_j$ are the same is given by $\left[\min_{0\leq\theta\leq359}\left\{\rho^{-}_{r^{\theta}_{R_j}}\right\}, \max_{0\leq\theta\leq359}\left\{\rho^{+}_{r^{\theta}_{R_j}}\right\}\right]$.

Fig. **3** shows the digital system to classify grey-scale images of $n \times n$ pixels. The images can present different position, rotation, noise and non-homogeneous illumination. In addition, if the object in the problem image has lack of data until a 15%, the system follows identifies it.

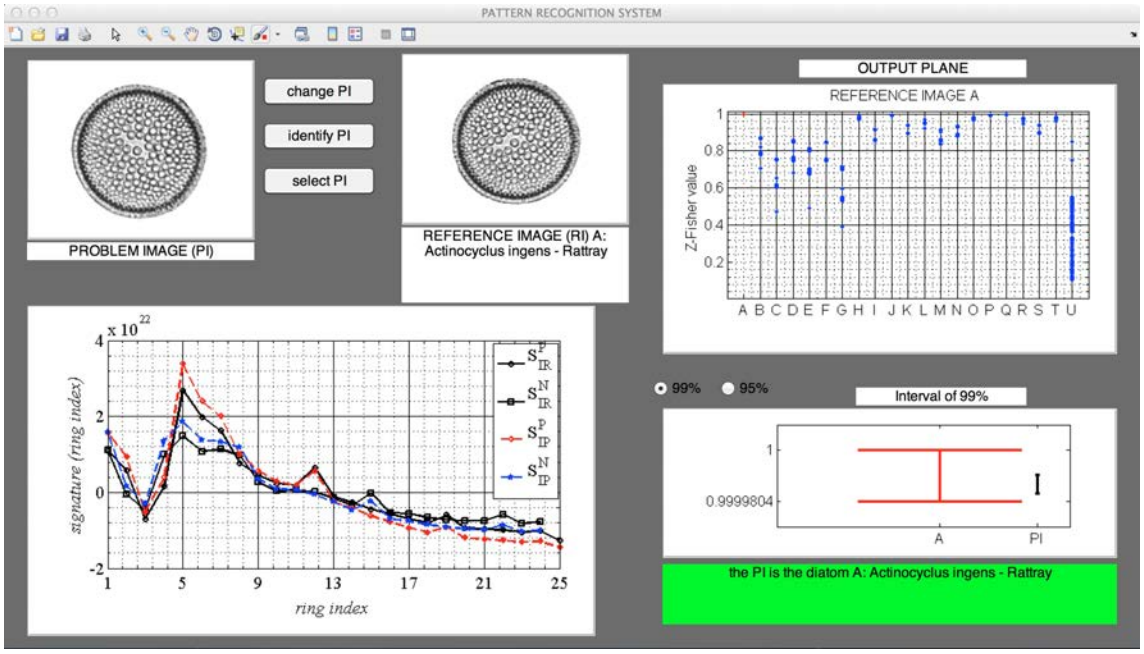


**Figure 3**: Pattern recognition digital system.

## 4. Conclusions

The digital system presented in this work has an excellent performance in the classification of grey-scale images. The image to be identified could have a translation and rotation of the object, noise and non-homogenous illumination. Moreover, if some data in the object are eliminated or overlapping by other object, the system follows recognizing its pattern.

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